Geotechnical Reliability of Levees

by

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ABSTRACT

The Corps of Engineers now performs cost-benefit analyses in a probabilistic framework. In support of such studies, geotechnical engineers must quantify the reliability of levees and other earth structures. Resource constraints for planning-level studies require that methods used permit the use of existing computer programs, be easy to implement in practice, and be useful where data are limited.

This paper reviews past Corps' guidance for assessing the geotechnical reliability of existing levees and reports the results of a research study to develop an improved and more comprehensive approach. In the developed methodology, several modes of levee performance are analyzed using a probabilistic capacity-demand model. By replicate analyses at different water heights, a conditional-probability of failure function for each mode can be developed as a function of flood water elevation. These in turn can be combined to develop a composite probability-of-failure function. Examples are provided for slope stability and underseepage, and other modes are discussed. The change in reliability for a levee subjected to increasing water heights is illustrated.

Based on the research, a new Engineering Circular (EC) is under development to implement the methodology in the Corps. However, there are still a number of known limitations for which additional research and development appear warranted. These are discussed.

INTRODUCTION

When the Corps of Engineers proposes construction of new levees or improvement of existing levees (typically raising the height), economic studies are required to assess the benefits and costs. Where an existing levee is present, the project benefits accrue from the increase in the degree of protection. Economic assessment of the improvement in turn requires an engineering determination of the probable level of protection afforded by the existing levee.

A research project by the author (Wolff, 1994) at Michigan State University involved developing and testing procedures that can be used by geotechnical engineers to assign conditional probabilities of failure for existing levees as functions of flood water elevation. Such functions may be used by economists to estimate benefits from proposed levee improvements. More recently, the author, under contract with Shannon and Wilson, Inc., to the Corps, prepared a draft Engineering Circular (EC) titled *Risk-Based Analysis in Geotechnical Engineering for Support of Planning Studies* (U.S. Army, 1997), which is in press at the time of this conference. It includes two appendices; Appendix A is entitled *An Overview of Probabilistic Analysis for*

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Geotechnical Engineering Problems (Wolff and Shannon and Wilson, 1997). Appendix B is the full text of the research report (Wolff, 1994) discussed above.

The new EC and its two appendices provide the current guidance for reliability assessment of levees in support of planning studies. This paper summarizes the recommended methodology, the research leading to it, and some remaining shortcomings that warrant further study.

EARLIER PRACTICE FOR EVALATING EXISTING LEVEES

Prior to 1991, existing levees that had not been designed or constructed to Corps' standards were often considered to be non-existent in economic analysis or to afford protection to some low and rather arbitrary elevation. (ETL 1110-2-328, U.S. Army, 1992) These assumptions are no longer permitted; in guidance issued in 1991-92, an existing levee is considered to afford protection with some associated probability.

Probable Failure and Non-Failure Points. Policy Guidance Letter No. 26 (U.S. Army, 1991) and draft ETL 1110-2-328, *Stability Evaluation of Existing Levees for Benefit Determination* (U.S. Army, 1992) provided simplistic quantitative guidance for assessing geotechnical reliability of existing levees. PGL No. 26 introduced the concept of levee reliability as a function of floodwater elevation, and introduced the concepts of *probable failure point* and *probable non-failure point*:

> *...commands...(i.e. Corps district and division offices) making reliability determinations should gather information to enable them to identify two points... The highest vertical elevation on the levee such that it is highly likely that the levee would not fail if the water surface would reach this level... shall be referred to as the Probable Non-Failure Point (PNP)... The lowest vertical elevation on the levee such that it is highly likely that the levee would fail... shall be referred to as the Probable Failure Point (PFP).. As used here, "highly likely" means 85+ percent confidence...*

PGL No. 26 went on to state:

> *If the form of the probability distribution is not known, a linear relationship as shown in the enclosed example, is an acceptable approach for calculating the benefits associated with the existing levees.*

PGL No. 26 took the probability of failure to increase linearly with flood water height from 0.15 at the PNP to 0.85 at the PFP. This assumption would permit an economist, in the absence of any further engineering analysis, to quantify reliability as a linear function. The engineer needs only, by some means, to identify flood water elevations for which the levee is considered 15 and 85 percent reliable.

Shape of Reliability Function. The assumption of linearity is expedient, and is the least-biased assumption where only two points are known and no other information is present.

However, the assumption of linearity may or may not be acceptable once some additional information *is* known. One of the objectives of the research was to determine what is in fact a reasonable function shape based on the results of some engineering analyses for typical levee cross sections and typical parameter values.

The Template Method. In ETL 1110-2-328, the *template method* was presented for determining PNP and the PFP. In this method, stated to be applicable only to levee cross-sections that have met other requirements of geometry, seepage, and slope stability, two idealized cross-sections, considered to meet desirable and minimal design standards, are drawn and fit within the cross-section of the existing levee. When the templates are matched to the existing cross-section at the toe points, the tops of these two templates are taken to be the PNP and PFP, respectively.

The template method for determination of the PNP for a "typical" clay levee by the ETL is illustrated in Figure 1. For a typical sand levee, the template crown would be widened to 12' and the side slopes flattened to 1v on 4h. The template method for determination of the PFP for a typical clay levee is shown in Figure 2. For a typical sand levee, the template crown would be widened to 8' and the side slopes flattened to 1v on 3h.

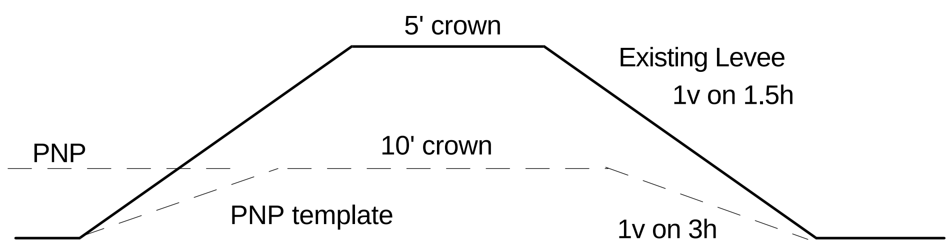

Figure 1. Template for PNP - clay levee

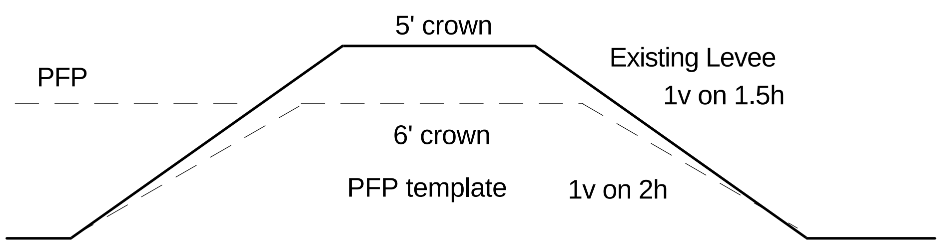

Figure 2. Template for PFP - clay levee

Implied Assumptions Regarding Slope Stability**.** When the definitions of the PNP and PFP are considered in conjunction with the template method, two significant assumptions are implied:

1) The PNP template, defined to be "*representative of a stable levee section for the soils involved and having an appropriate crest width and side slopes*", is implied to have a reliability of 85% and a probability of failure 15%.

2) The PFP template, defined to be a reduced section at which a levee would be stable for reduced periods of time, is implied to have a reliability of 15% and a probability of failure of 85%.

Reasonableness of these Assumptions**.** The Pr(f) = 0.15 associated with the elevation of the PNP is unreasonably high. Given this probability value, about 1 in 6 new levees built to 1V on 3H slopes would be expected to fail. Various studies (e.g. Wolff, 1985; Shannon and Wilson, 1994, Vrouwenvelder, 1987) indicate that dams, levees and dikes designed to Corps criteria or Dutch criteria would be expected to have probabilities of failure on the order of 10^{-3}, 10^{-4}, and even lower. Hence, the conditional probability of failure associated with the PNP elevation determined from the template method should be expected to be in the range of perhaps 0.001 to 0.0001, not 0.15.

The Pr(f) = 0.85 associated with the elevation of the PFP also appears to be high in the context of experience, although probably not so much as for the PNP template. If an engineer judged this section equally likely to fail as to stand up, which seams to be a reasonable assumption, the section would correspond to a $Pr_f = 0.50$ rather than 0.85.

As slope stability has a well-developed mathematical basis, and is relatable to measurable soil properties, it is a candidate for inclusion in a probabilistic levee reliability methodology.

Performance modes other than slope stability**.** The template method was the only procedure sufficiently quantified to permit assigning probability values, and it presumably relates primarily to slope stability. Other performance modes were required to be considered, but no quantitative methods to do so were presented. Other potential performance modes include:

1) Safety against **overtopping**, including flood duration and ability of levee materials to endure that duration.

2) Safety against **underseepage** with associated sand boils and piping. This is a well-recognized hazard not even considered in the template method. For underseepage, safety is essentially independent of crown width and slopes; but is highly dependent on foundation stratigraphy.

3) Safety against **through-seepage** and associated internal erosion, piping, or surface erosion of the landside slope (cited in PGL 26). This mode *is* related to the levee template and material; however equating of the PNP and PFP levels to Pr(f) = 0.15 and 0.85 does not directly follow from any through-seepage considerations.

4) Safety against **surface erosion** of slopes and crest **resulting from rainfall** (cited in PGL 26). This is primarily related to slope, material type, and vegetative cover. The PNP and PFP are not directly related to these factors.

5) Safety against **surface erosion due to current and wave attack on the riverside slope** (not specifically cited in PGL 26). During high stages when the upper part of the riverside slope is exposed to attack, current velocities are higher, and fetch distances are longer.

6) **Flood duration.** Some levees may be subjected to significant water heights for many months. When this occurs, the phreatic surface within the levee will rise, increasing pore pressures and increasing the risk of failure due to through-seepage, underseepage and slope stability. This is acknowledged in a rudimentary way in the draft ETL which reduces the crest width when the levee is exposed to flood heights for only a limited time.

7) **Geometry beyond the levee toe**, such as distance to the river, location and depth of borrow areas, and presence or absence of vegetation and tree cover between the levee. This is not considered in the template method. These conditions may impact slope stability, underseepage, current velocities, and wave fetch distance.

8) **Other items** from the preliminary inspection, such as "*vegetation ... animal burrows, man-made excavation through surface impervious layers, ...cracks, toe-undercutting, slides, and ...soil creep "* are to be considered in developing the function, but no guidance is provided as to how to do so.

THE CONDITIONAL PROBABILITY OF FAILURE FUNCTION

The research (Wolff, 1994) and the forthcoming EC take the approach of constructing a conditional probability of failure function dependent on flood water elevation. A number of performance modes are considered, a separate function is developed relating the conditional probability of failure for each mode to flood water elevation, and these are then combined.

The conditional probability of failure can be written as:

$$\Pr(f) = \Pr(\text{failure} \mid \text{FWE}) = f(\text{FWE}, X_1, X_2, \ldots X_n) \tag{1}$$

In the above expression, the symbol "|" is read *given* and the variable FWE is the flood water elevation. The random variables X_1 through X_n denote relevant parameters such as soil strength, permeability, top stratum thickness, etc. Equation 1 can be restated as follows: "The probability of failure, given the flood water elevation, is a function of the flood water elevation and other random variables."

Two extreme values of the function can be readily estimated by engineering judgment. For flood water at the same level as the landside toe (base elevation) of the levee, $P_f = 0$; for flood water at or near the levee crown (top elevation), $P_f \rightarrow 1.00$. The question of primary interest, however, is the shape of the function between these extremes. Quantifying this shape is the focus herein; how reliable might the levee be for, say, a ten or twenty-year flood event that reaches half or three-quarters the height of the levee?

Reliability (R) is defined as:

$$R = 1 - P_f \qquad (2)$$

hence, for any flood water elevation, the probability of failure and reliability must sum to unity. For flood water part way up a levee, R could be near zero or near unity, depending on factors such as levee geometry, soil strength and permeability, foundation stratigraphy, etc. Five possible shapes of the R = f(FWE) function are illustrated in Figure 3. For a "good" levee, the probability of failure may remain low and the reliability remain high until the flood water elevation is rather high. In contrast, a "poor" levee may experience greatly reduced reliability when subjected to even a small flood head. It is hypothesized that some real levees may follow the highlighted intermediate curve, which is similar in shape to the "good" case for small floods, but reverses to approach the "poor" case for floods of significant height. Finally, a straight line function is shown, similar to the previously-assumed linear relation between reliability and flood height.

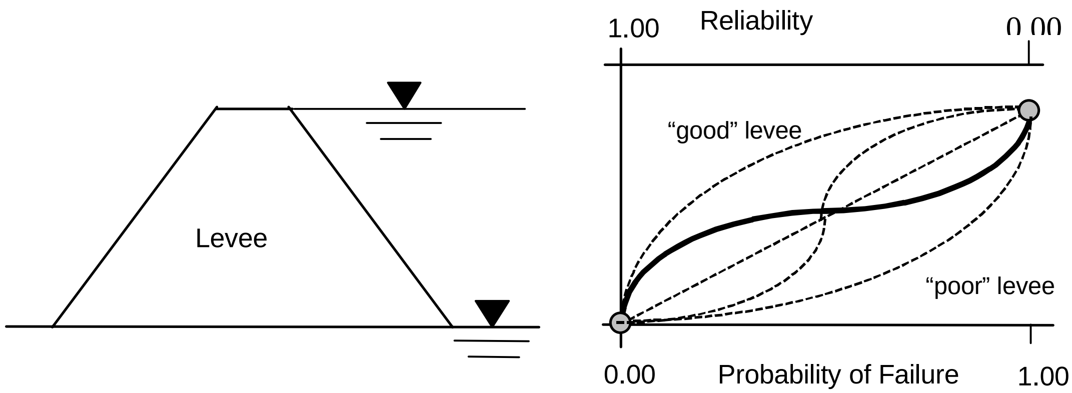

Figure 3. Possible Reliability vs. Flood Water Elevation Functions for Existing Levees

RELATED RELIABILITY ANALYSIS PROCEDURES USED BY THE CORPS

Quantifying geotechnical reliability for planning studies poses a challenge. Many published techniques are too complex for routine practice, require more data than will be available, or require specialized computer programs. Given these constraints, the selected probabilistic methods must be based on some combination of limited testing and experience, and existing procedures and computer programs (e.g. for slope stability and seepage analysis) must be used as much as feasible.

The procedures for constructing conditional Pr(f) functions are built on earlier-developed methodology for navigation structures. Several studies have been made to develop procedures (Wolff and Wang, 1992a, 1992b; Shannon and Wilson and Wolff, 1994) and to promulgate guidance (U.S. Army, 1992a). In general, these methods are based on expressing the uncertainty in structural performance as a function of the uncertainty in the values of the variables in an associated performance model, such as a slope stability or underseepage analysis.

GENERAL METHODOLOGY

The term *probability of unsatisfactory performance*, Pr(U) is often used in Corps reliability guidance (U.S. Army, 1992a,b) in lieu of the more common *probability of failure* Pr(f), to reflect the fact that remedial measures are expected to be taken before a catastrophic failure condition becomes imminent. However, for existing levees, the latter term may be accurate. In economic risk assessments, Pr(U) or Pr(f) values for several performance modes are combined with economic consequences (flooding, loss of service, etc.) to determine probabilistic benefits and costs. Ideally, one would like to obtain "absolute" values for Pr(U) or Pr(f). However, several factors restrict the task to calculating comparative measures. These include limited data, lack of knowledge regarding the shape of probability distributions, and the use of approximations such as first-order second-moment (FOSM) methods, which facilitate the use of existing computer programs.

<u>Determining the Reliability Index</u>**.** The basic scheme for reliability analysis is summarized in Corps' guidance (U.S. Army, 1992a,b) and is only briefly reviewed here. Comparative reliability is measured by the *reliability index* β. As illustrated in Figure 4, β is the number of standard deviations by which the expected value of the *performance function* exceeds the *limit state.* The natural log of the factor of safety, ln FS, is taken as the performance function and the condition ln FS = 0 is taken as the limit state. β incorporates the information inherent in the factor of safety, but additionally provides a measure of the relative certainty or uncertainty regarding parameter values. Calculating β involves five steps:

1) Identifying a performance function and limit state, typically ln FS = 0.
2) Identifying the *random variables* contributing uncertainty.
3) Characterizing the random variables by of their expected values E[X], coefficients of variation V_x and, where necessary, their correlation coefficients $\rho_{X,Y}$.
4) Determining the expected value and standard deviation of the performance function using the Taylor's Series Finite Difference (TSFD) method.
5) Evaluating β from the results of step 4.

For step 1, the safety factor against slope failure is commonly determined using the UTEXAS computer program (Edris and Wright, 1987). For underseepage, the factor of safety is taken as the ratio of the critical gradient i_c to the exit gradient i_o at the landside toe (U.S. Army, 1956). Exit gradients may be calculated by hand solution, spreadsheet, or with the program LEVEEMSU (Wolff, 1989). For other performance modes, widely-accepted performance functions and limit may not be available to the same extent as for slope stability and underseepage; additional research may be required.

For step 2, random variables for slope stability are typically the shear strength parameters c and ϕ. For underseepage analysis they are typically the horizontal permeability of pervious substratum foundation materials k_f, the vertical permeability of semipervious top blanket materials $k_{b,}$, and the thickness of the top blanket at the landside levee toe, z.

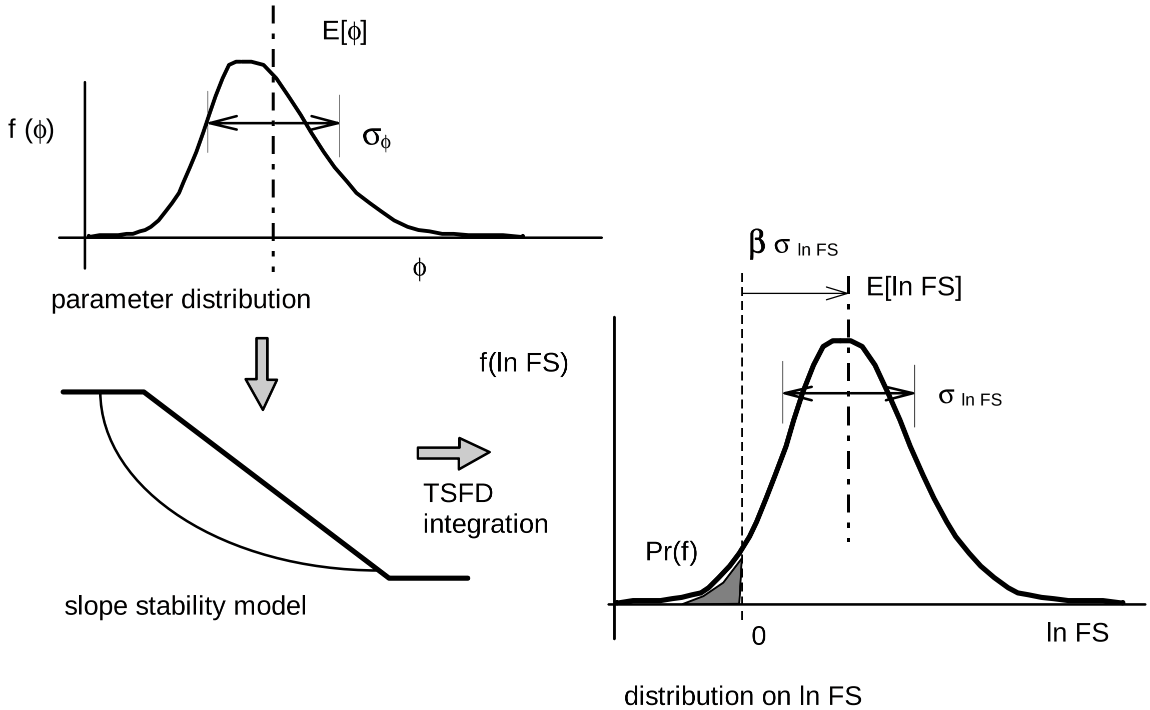

Figure 4. Probability of Failure, Reliability Index, and Method of Moments

For step 3, where sufficient data are available, the probabilistic moments may be calculated by standard statistical means. However, for many existing structures, they must be assigned from limited data and judgment based on similar structures. The standard deviation can be obtained by multiplying the expected value by an estimated coefficient of variation, based on a limited but growing body of data. Where no data are available, values can often be estimated by taking the engineers' judgment regarding reasonable parameter limits as corresponding to the expected value plus and minus 2.5 or 3.0 standard deviations.

For step 4, the moments of the performance function are estimated from the moments of the random variables. E[FS] can be approximated using the Taylor's series first-order, second-moment (FOSM) mean value approach as:

$$E[FS] = FS(E[X_1], E[X_2]\ldots E[X_n]) \tag{3}$$

where X_i represents the random variables such as c, ϕ, k_f, k_b, or z. In other words, the expected value of the performance function is taken as the value of the function evaluated at the expected values of the random variables.

Continuing with the Taylor's series approach, the standard deviation of the factor of safety is the square root of the variance of the factor of safety, which is calculated as

$$\mathrm{Var[FS]} = \sum\left(\frac{\partial FS}{\partial X_i}\right)^2 \sigma_{X_i}^2 + 2\sum\left(\frac{\partial FS}{X_i}\frac{\partial FS}{X_j}\right)\rho_{X_i,X_j}\sigma_{X_i}\sigma_{X_j} \tag{4}$$

Where random variables are taken to be independent, the second summation drops out.

The partial derivatives are calculated at the expected value of each random variable. More sophisticated methods have been proposed, such as Hasofer and Lind's (1977) method, wherein the Taylor's series is expanded about an unknown "failure point" by successive iteration. This has the advantage of providing invariant solutions; however, its computational complexity presently limits its practicality for planning-level studies; each evaluation of a performance function requires a computer run, and the method requires considerable iteration. Using existing programs, the partial derivatives in Equation 4 may be estimated numerically, using finite differences, as

$$\frac{\partial FS}{\partial X_i} \approx \frac{FS(X_{i+}) - FS(X_{i-})}{X_{i+} - X_{i-}} \tag{5}$$

where X_{i+} and X_{i-} represent the random variable X_i taken at some increment above and below the expected value. Although a very small increment would give the most accurate value, Corps' practice has been to take the increment at $\pm$ 1 σ from the expected value. This large increment picks up some of the behavior of nonlinear functions over their most probable range, and leads to computational simplicity. With this increment and independent random variables, Equation 4 becomes:

$$\mathrm{Var[FS]} \approx \sum_{i=1}^{n}\left(\frac{FS(X_{i+}) - FS(X_{i-})}{2}\right)^2 \tag{6}$$

Finally, in step 5, β is calculated as previously shown in Figure 4:

$$\beta = \frac{E[\ln FS]}{\sigma_{\ln FS}} \tag{7}$$

The required probabilistic moments for ln FS are determined from the moments for FS as:

$$\sigma_{\ln FS} = \sqrt{\ln\left(1 + V_{FS}^2\right)} \tag{8}$$

$$E[\ln FS] = \ln\left(E[FS]\right) - \frac{\sigma_{\ln FS}^2}{2} \tag{9}$$

Although not absolute measures of reliability, β values provide consistent comparisons across performance modes and across structures. They permit comparing the relative reliability of one structure to another, the relative reliability of a structure for different performance modes such as slope failure and seepage failure, and the relative change in reliability of a structure subjected to changing loads, such as a levee embankment subjected to rising water levels.

Estimating Pr(f). With comparative reliability expressed as β, planners have a means compare the relative need for remedial work among several structures or components. Nevertheless, probability values Pr(f) are often desired as multipliers for the economic consequences of adverse performance. In this case, ln FS is *assumed* normally distributed and Pr(f) is taken as the cumulative probability for the standard normal distribution evaluated at -β standard deviations:

$$\Pr(U) = \Phi(-\beta) \tag{10}$$

While these are not precise probability values, due to the numerous assumptions, the resulting expected costs of alternatives are considered to provide valid comparisons.

EXAMPLE PROBLEMS

To investigate the relationships between Pr(f) and flood height, two example problems were analyzed in the research. Figure 5 shows one of these, a pervious sand levee overlying a thin clay top blanket which in turn overlies a thick pervious sand substratum. This section, although deliberately made steep and pervious to illustrate the change in Pr(f) with flood height, is not unlike some private levees along the upper Mississippi and Illinois Rivers. The second example was a clay levee on a clay top blanket with irregular geometry.

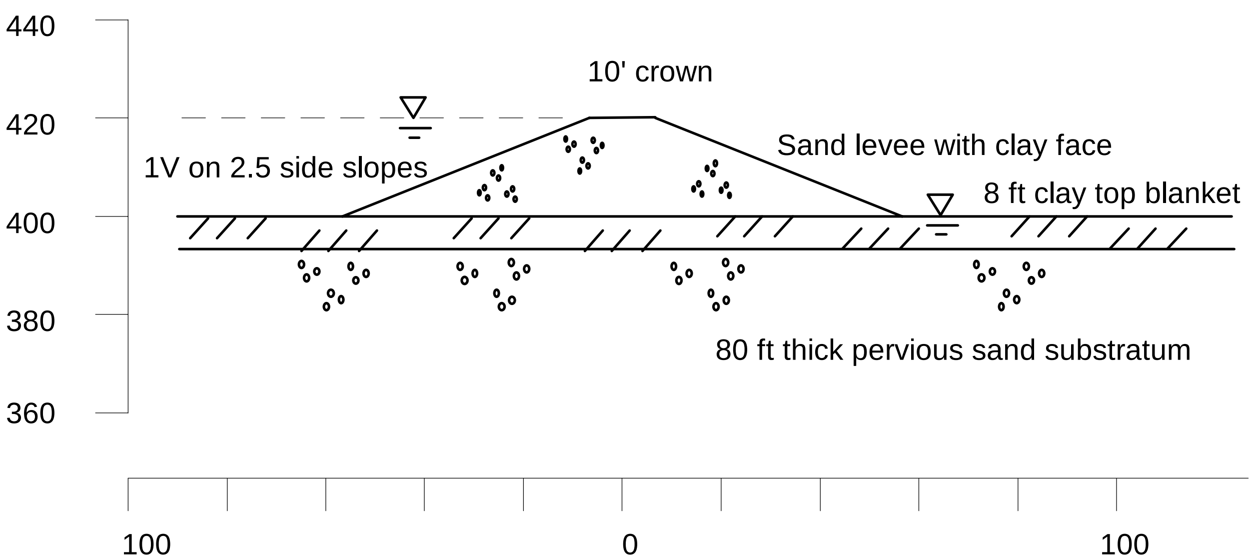

Figure 5. Cross-Section for Pervious Sand Levee Example

EXAMPLE UNDERSEEPAGE ANALYSIS

Using the methodology previously described, probabilistic underseepage analyses were performed for flood elevations ranging from el. 400, the natural ground surface, to el. 420, the levee crown. Random variables were characterized as shown in Table 1. Using the permeability values for the top blanket k_b and k_f, the moments of a new random variable, their ratio, was calculated using the TSFD method. E[FS] and var[FS] are calculated using Equation 6 as shown in Table 2 for the highest water elevation. These were used to calculate β, and converted to Pr(f) using Equation 10. The resulting function relating the conditional probability of underseepage failure to flood height is shown in Figure 6. The function is S-shaped, and Pr(f) is low for floodwater heights less than about one-half the levee height, even for an assumed cross-section intended to represent potentially deficient conditions.

Table 1
Random Variables for Underseepage Analysis, Sand Levee Example

Parameter	Expected Value	Standard Deviation	Coefficient of Variation
kf	0.1 cm/s	0.03 cm/s	30%
kb	1 x 10-4 cm/s	0.3 x 10-4 cm/s	30%
z	8.0 ft	2.0 ft	25%
d	80 ft	5 ft	6.25%

Table 2
Probabilistic Underseepage Analysis for Water at Elevation 420. (H = 20. ft)

Run	k_f/k_b	z	d	h_o	i	Variance	Percent of total Variance
1	1000	8.0	80.0	9.357	**1.170**		
2	600	8.0	80.0	9.185	1.148		
3	1400	8.0	80.0	9.451	1.181	0.000276	0.30
4	1000	6.0	80.0	9.265	1.544		
5	1000	10.0	80.0	9.421	0.942	0.090606	99.69
6	1000	8.0	75.0	9.337	1.167		
7	1000	8.0	85.0	9.375	1.172	0.000006	0.01
				Total		0.090888	100.0

The shape can be understood by reviewing Figure 5. At low flood heights, the normal curve representing ln FS is well to the right of the limit state. As the flood height increases, E[ln FS] decreases and the curve moves to the left, but $V_{\ln FS}$ tends to stay constant, keeping the width of the curve constant. The area under the curve below the limit state (i.e., Pr(f)) increases at an increasing rate, beginning at about 10 ft of head in the example. Once E[ln FS] drops below 0.0, occurring at about 15 ft of head in the example, the peak of the normal curve has moved below the limit state. Increasing heads continue to increase Pr(f), now in excess of 50%, but at a decreasing rate. The shape of the curve is also consistent with observations during floods; even substandard levee sections often perform adequately for low head conditions, but performance can deteriorate rapidly as water levels increase.

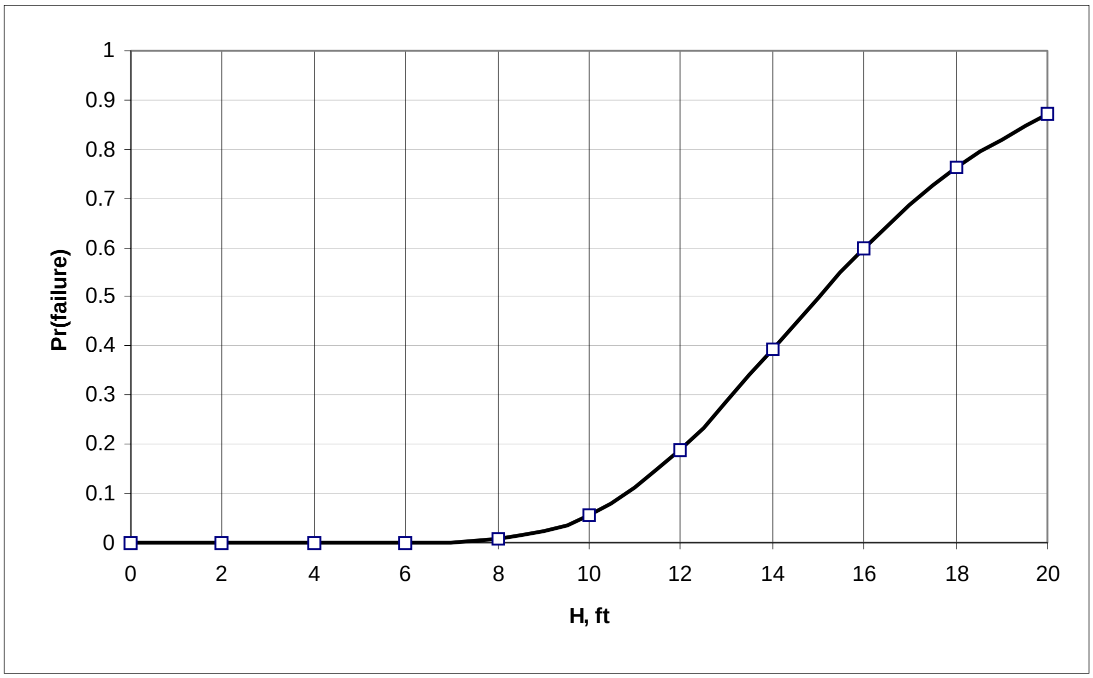

Figure 6. Probability of Underseepage Failure vs. Floodwater Elevation

EXAMPLE SLOPE STABILITY ANALYSIS

Slope stability analyses were performed for the sand levee example in Figure 5 for a range of water levels using UTEXAS2. Random variables were characterized as shown in Table 3. Two distinct piezometric surfaces were modeled in the two sand materials. The piezometric surface in the embankment was approximated as a straight line from the point where the flood water intersects the riverside slope to the landside levee toe. The piezometric surface in the foundation was obtained from the expected value condition in the underseepage analyses. This results in a piezometric surface in the foundation that is above the natural ground on the landside of the levee. Additional refinement could be made by making this piezometric surface a random variable.

Table 3
Random Variables for Slope Stability Analysis, Levee Reliability Example

Parameter	Expected Value	Standard Deviation	Coefficient of Variation
ϕ (embankment sand)	30 deg	2 deg	6.7%
s_u (clay foundation)	800 lb/ft^2	320 lb/ft^2	40%
ϕ (foundation sand)	34 deg	2 deg	5.9%

Changing strength parameters in the probabilistic analysis and changing piezometric surfaces as the water level increases both lead to changes in the location of the critical surface. With flood water to elevation 410, critical surfaces occur both in the foundation clay and near the surface of the embankment (Figure 7). As the water level increases and piezometric levels rise in the sand embankment, the critical surfaces all move to the embankment.

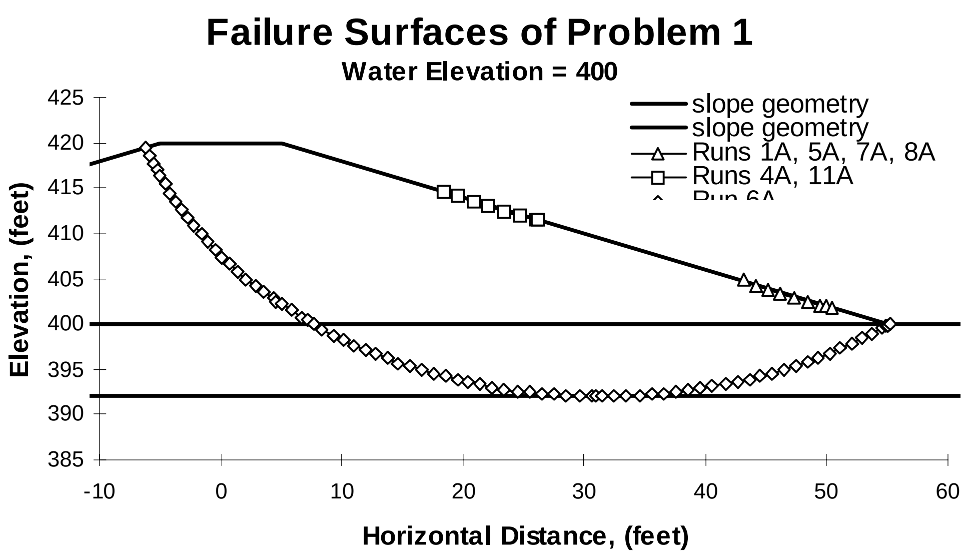

Figure 7. Critical Slip Surfaces for Floodwater to Mid-Height of Levee

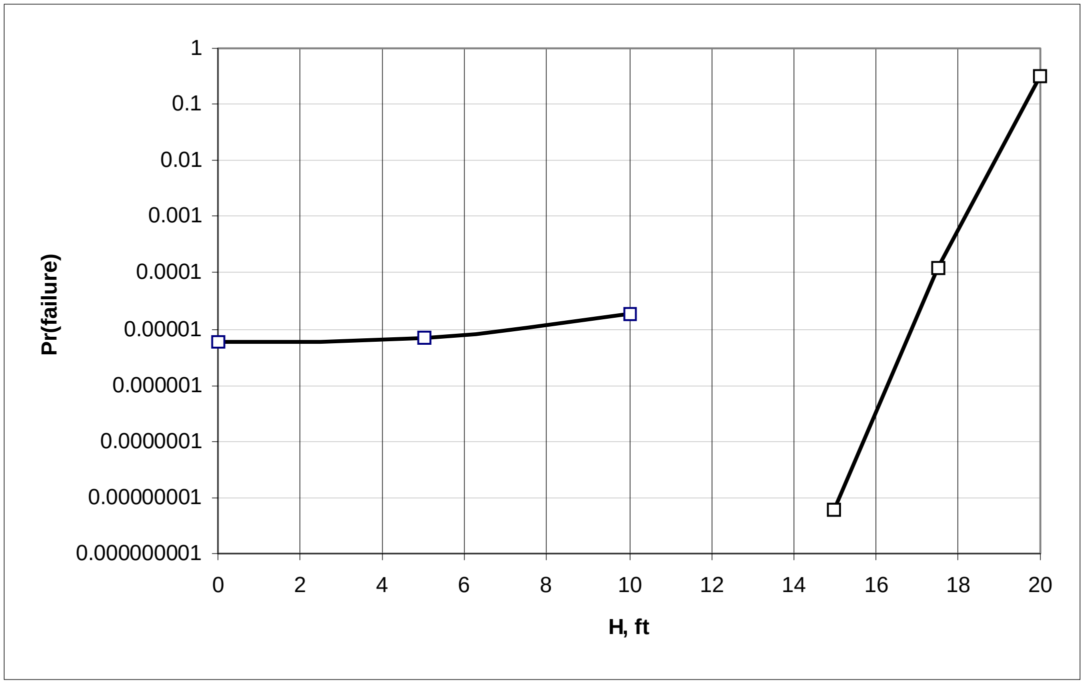

Figure 8. Probability of Slope Failure vs. Floodwater Height

The resulting conditional Pr(f) function for slope stability is shown in Figure 8. It is observed that Pr(f) is almost negligible until the flood water reaches about three-quarters the levee height, a point where the piezometric surface in the embankment begins to significantly affect the stability of potential shallow failure surfaces on the landside slope.

A discontinuity in Pr(f) is observed as the flood height is increased from 10 ft to 15 ft, Pr(f) abruptly decreases, then begins to rise again. This illustrates an interesting facet of probability analysis; Pr(f) is a function not only of the expected values of the factor of safety and the underlying parameters, but also of their coefficients of variation. In the present case, at a flood height between 10 and 15 ft, some of the critical surfaces move from the foundation clay, with a high coefficient of variation for its strength, to the embankment sands, for which the coefficient of variation is smaller. This decreases β and Pr(f). Even though the safety factor may decrease as the flood height increases, if the value of the smaller safety factor is more certain due to the lesser strength uncertainty, Pr(f) may decrease.

OTHER PERFORMANCE MODES

The curves in Figures 6 and 8 illustrate the conditional Pr(f) for only two failure modes. Other modes of potentially adverse performance include internal erosion from through-seepage, and external erosion due to seepage exit, current velocity, and wave attack. Preliminary approaches to analysis of some of these conditions are suggested in the research report (Wolff, 1994) and numbers are calculated for illustration; however, performance functions and limit states for these modes are not nearly so well developed and accepted as those for slope stability and underseepage.

BUILDING THE COMPOSITE FUNCTION

Where Pr(f) versus flood height functions can be developed for each possible performance mode, and where modes can be assumed independent, a total Pr(f) function can be developed by combining the probabilities as a series system. For an independent series system, the overall reliability R is given by

$$R = R_1 R_2 \ldots . R_n \tag{11}$$

Applying Equation 11 at a series of flood water elevations gives:

$$R(FWE) = R_1(FWE)\, R_2\,(FWE) \ldots R_n\,(FWE)$$

(12)

Where modes have some correlation, as is likely the case for seepage and slope stability, the assumption of independence is conservative and leads to an upper bound on the probability.

Figure 9 shows the combined conditional probability-of-failure function for the sand levee example. The functions for underseepage and slope stability have previously been discussed. The function for through-seepage was developed using a modification of Rock Island District design criteria for sand levees. The function for surface erosion was developed by assuming a critical scour velocity and comparing it to the river velocity using a simple Manning equation approach; more sophisticated models can undoubtedly be constructed using the Corps' HEC models. Finally, the "judgment" curve represents the probability values that can be assigned by the engineer for items not explicity modeled, such as observed cracks and animal burrows. Techniques to assign and calibrate such values require further study.

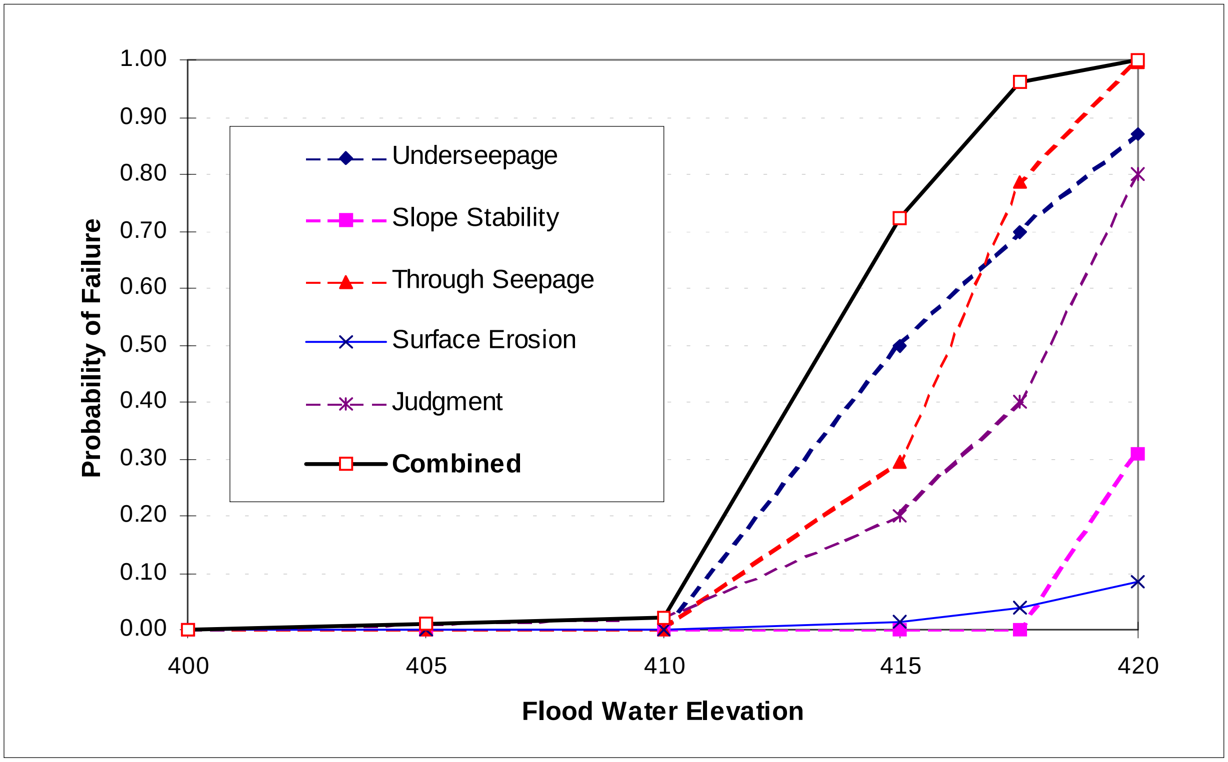

Figure 9. Combined Conditional Probability of Failure Function

REMAINING LIMITATIONS / AREAS FOR FURTHER STUDY

The illustrated methodology provides an important step to developing reliability functions that include site-specific information regarding soil conditions along a levee; nevertheless, there are many remaining limitations, and areas for further research. These are summarized in Appendix A to the forthcoming EC (Wolff and Shannon and Wilson, 1997). They include:

1. Varying interpretations regarding the interpretation of probabilistic slope stability analysis. A slope is a system of an infinite number of possible failure surfaces. As the critical surface in deterministic analysis does not in general, coincide with that for probabilistic analysis, a number of approaches can be developed which yield different solutions.

2. Application of spatial correlation theory to soil parameters. As soil is a continuous medium, the appropriate characterization of uncertainty in a two-dimensional slope stability or seepage analysis is dependent on the size of the modeled area and free body.
3. Application of spatial correlation theory to long earth structures. Similarly, real levees may be many miles in length. Intuitively, a long levee is less reliable than a

replicate shorter one. Elegant mathematical solutions are available to treat this problem, however, appropriate values to use in such models remain problematical

SUMMARY AND CONCLUSIONS

Planning studies for rehabilitation of Corps' projects now require quantifying the reliability of embankments and other engineering features. Performing reliability analyses of existing structures given resource constraints requires adapting probabilistic methods to use existing computer programs and developing some simple approaches that can be used where little or no test data are available. The reliability index concept, wherein uncertainty in performance is related to the uncertainty in underlying random variables, is gaining application for Corps' studies, and is a convenient approach for assessing levee reliability.

Given that Pr(f) can be calculated for different performance modes and different flood water elevations, these values can be combined to provide the desired conditional probability-of-failure functions; however, the underlying deterministic models for performance modes other than slope stability and underseepage warrant further study.

For levees subjected to increasing floodwater heights, the probability of failure versus floodwater height function is typically S-shaped. Probabilities of failure may be low at low heads, but reliability may deteriorate rapidly as flood water levels increase. This finding, supported mathematically herein, agrees with engineering intuition and observed behavior of levees during floods.

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